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The Application Of The Laplace Transform In The Heat Equation

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Abstract

The one dimensional of homogeneous heat equation is two ordinary partial differential equations which general form $\frac{?u}{?t} = k\frac{2^2 u}{?x^2}$ *, so needs solution. The one dimensional of homogeneous heat equation the method is ever been solved by Fourier series and Fourier Integrals. The other method used to in solving one dimensional of homogeneous heat equation is Laplace Transform, which is used to transform into ordinary differential equation is one dimension* $\frac{?u}{?t} = k\frac{?^2 u}{?x^2}, 0 < x < L, k > 0$ *for initial conditions* $s\bar{u}(x,s)\text{-}u(x,0) = \frac{d^2\bar{u}(x,s)}{dx^2}$*. Inverse general solution that satisfies boundary conditions* $u(0,t) = 0, u(L,t) = 0,$ *is found, for every t and initial conditions* $u(x,0) = \emptyset(x), 0 < x < L$ *that have been transform is* $u(x,t)$*. From the result, it an obtain temperature on the poin x in the solid at time t.*

Keywords : Equation, Fourier Integral, Ordinary partial equation, Laplace Transform

INTRODUCTION

Differential equations often appear in mathematical models on physical and geometric problems, involving at least two or more of the two variable differences. Differential equations are also of great importance in physics, biology and other fields, since many laws and physical relationships are in the form of mathematical modeling as differential equations. One of the second-order partial differential equations used in mathematical physics applications is the heat equation equation in three dimensions that takes the form of (AF 2018).

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), u = u(x,y,t), k > 0$$

Where u is the temperature in a solid and constant $k = \frac{K}{CP}$ is the diffusivity, where K is the thermal conductivity, c is specific heat and ρ is density considered a constant.

As for several studies that have been carried out by several researchers, based on (Sari 2022), partial differential equations (PDP) can be solved analytically and numerically. One of the analytical PDP solutions is to use the Laplace transform. This method is widely used to solve initial and boundary condition problems. In this article for PDP solutions using the Laplace transform. Solving the PDP using the Laplace transform is done by transforming the equation and substituting the initial value given so that it is obtained in the form of an ordinary differential equation (PDB). Furthermore, the Adomian Laplace decomposition method is one of the methods that can be used to solve differential equations that combine the Laplace transform method and the Adomian decomposition method. The solution to the addiction-diffusion equation is obtained by applying the Laplace transform to the addiction-diffusion equation (Ahmadi, Hartono, and Binatari 2017), substituting the initial conditions, expressing the solution in an infinite series, determining the terms, and applying the inverse Laplace transform to the terms term of the infinite series. The result of this paper is that the addiction-diffusion equation solution can be obtained by the Adomian Laplace decomposition method (Abdy, Wahyuni, and Awaliyah 2022).

Heat transfer is a process that occurs because of the flow of heat in a system. This process can be explained through the heat equation which is a fairly complex non-linear second order partial differential equation. The heat equation is quite complicated to solve analytically, but it can also be solved numerically (Mubaroq and Saptaningrum 2022). In (Jahroo Pratiwi 2023) several more real cases, partial differential equations, including the Laplace equation, are difficult to solve exactly, so numerical methods are used as an alternative. One of the numerical methods that is often used to solve boundary value problems in a partial differential equation is the Boundary Element Method (MEB). In this method the domain boundary is partitioned into a finite number of line segments which are then used to evaluate the integral boundary equation. In this paper MEB is implemented in solving Laplace's equation with mixed boundary conditions, namely Dirichlet boundary conditions and Neumann boundary conditions

Further research shows that the Naive Bayes method has drawbacks. If there are events that are not in the training data, then there is a possibility that there is a predicted value whose probability is zero (Harizahayu 2020). This makes the predicted probability value less accurate. To overcome this problem the author adds the Laplace Smoothing method. This application will later be built on the Android Studio platform. Specifically, in each iteration, the algorithm selects a target for the next sampled value based on the current sampled value (S 2020). Based on the dimensions of the heat equation consists of three, namely one dimension, two dimensions, and three dimensions. The heat equation is used to determine the temperature of each point on the cylindrical length limit. In determining the solution for u or solving the heat equation, it must meet the conditions determined by the physical system, namely the initial conditions and boundary conditions.

The one-dimensional homogeneous heat equation has been solved with the Fourier series and the Fourier integral by Kreyszig. One of the solutions to the second-order partial differential equations is by using the Laplace transform, because the heat equation is a second-order partial differential equation, it is interesting to solve with the aplace transform. The Laplace transform is used to provide another alternative in solving one-dimensional homogeneous heat equations. In this study the application of the Laplace transform can solve one-dimensional homogeneous heat equations that meet the initial and boundary conditions. Where in this study also limits the problem in the one-dimensional homogeneous heat problem in the form $\frac{?u}{?t} = k\frac{?^2u}{?x^2}, 0 < x < L, k > 0$ for initial conditions $u(x,0) = \emptyset(x)$ and boundary conditions $u(0,t) = 0, u(L,t) = 0,$ or the horizontal with a constant temperature at every t. Based on the literature study, the authors were able to solve the one-dimensional homogeneous heat equation using the Laplace transform with the research title: "The Application of the Laplace Transform in the Heat Equation"

RESEARCH METHODS

This research is a literature study by taking study materials and references from Google Scholar which is research related to this research. The steps taken by the author in this study are as follows:

1. Given a one-dimensional homogeneous heat equation with initial conditions boundary conditions

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2. Use the Laplace transform on the one-dimensional homogeneous heat equation to become an ordinary differential equation
3. Solve step 2 to obtain a general solution
4. Transform the given terms and substitution to the general solution in step 3
5. Take the inverse Laplace transform in step 4 so that a general solution is obtained in determining the temperature.

RESULTS AND DISCUSSION

The general function of temperature in the solid is a solution to a differential equation called the heat equation. The heat equation can be derived directly using Fourier's law from the heat transfer to a horizontal cylindrical $L > 0$ each $0 = x = L$ and $t = 0$. If the heat (Q) t at time in the solid is between x and $x+h$, if A is the cross-sectional area, then the volume of the solid is Ah , the mass of the solid is (Δm) is ρAh . For more details see the picture below

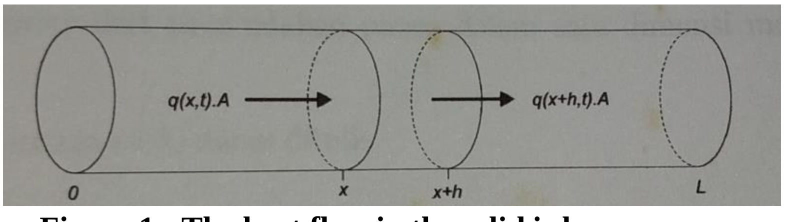

Figure 1 : The heat flow in the solid is homogeneous

Heat in the solid at t (time) is

$$Q(x,t,h) = c?mu(x,t) \qquad (1)$$

or

$$Q(x,t,h) = c?Au(x,t)h,$$

Rate change Q is

$$\frac{dQ}{dt} = c?A\frac{?u}{?t}h \qquad (2)$$

In the principle of energy conservation, the rate of change must be the same, the rate of heat entering is smaller than the rate of heat leaving so,

$$\frac{dQ}{dt} = q(x,t)\cdot A - q(x+h,t)\cdot A \qquad (3)$$

Substitution equation (2) into Equation (3),

$$c\rho\frac{\partial u}{\partial t} = -\frac{q(x+h,t)-q(x,t)}{h} \qquad (4)$$

if $h?0$, so the equation (4)

$$c?\frac{?u}{?t} = -\frac{?q}{?x} \qquad (5)$$

According to Fourier's law of heat transfer in one dimension $q = -K\left(\frac{?^2u}{?x^2}\right)$, equation (5) to be

$$\frac{?u}{?t} = k\frac{?^2u}{?x^2} \qquad (6)$$

where $u(x,t)$ is thermal solid x at t, $k = \frac{K}{c?}$ is diffusivity, K is conductivity thermal, c is specific heat and ? is density considered. In the equation, given satisfied boundary conditions from $x = 0$

and $x = L$ with temperature $u(0,t) = 0$, $u(L,t) = 0$, for all t. Distribution temperature at x in $t = 0$ is $?(x)$ called initial conditions in form $u(x,0) = ?(x), 0 < x < L$.

Laplace Transform Application In One-Dimensional Homogeneous Heat Equation

For determine the temperature of solid with constant for each solid, so find the solution of u at equation (6), which must also requirements by the physical system as described above is initial conditions and boundary conditions.

To solve one-dimensional homogeneous heat equations using the Laplace transform, the properties of the Laplace transform is :

1. Laplace Transform with t^n

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s) = (-1)^n F^n(s) \tag{7}$$

2. Laplace Transform from Integral

$$\mathcal{L}\left\{\int_0^t F(u)du\right\} = \frac{F(s)}{s} \tag{8}$$

The Laplace transform is used to solve a one-dimensional homogeneous heat equation that satisfies the initial and boundary conditions. The Laplace transform can reduce one-dimensional homogeneous heat equations to ordinary differential equations. The general form of the second order homogeneous heat equation which can be solved by transforming using the Laplace transform in equation (6), then equation (6) can be written

$$\mathcal{L}\left\{\frac{?u(x,t)}{?(t)}\right\} = k\mathcal{L}\left\{\frac{?^2u(x,t)}{?x^2}\right\} \tag{9}$$

where k is constanta. Transform equation (6) at , so

$$\mathcal{L}\left\{\frac{?u(x,t)}{?(t)}\right\} = \int_0^8 e^{-st}\frac{?u(x,t)}{?t}dt \tag{10}$$

For the solve equation (10) use the Parcial integral $wv - \int vdw$, that

$$\int_0^8 e^{-st}\frac{?u(x,t)}{?t}dt = e^{-st}u(x,t)|_0^8 + s\int_0^8\{e^{-st}u(x,t)dt\} \tag{11}$$

Get equation (10) is

$$\mathcal{L}\left\{\frac{\partial u(x,t)}{\partial t}\right\} = s\bar{u}(x,s) - u(x,0)$$

so

$$\mathcal{L}\left\{\frac{\partial u(x,t)}{\partial x}\right\} = \frac{d\bar{u}(x,t)}{dx}$$

that

$$\mathcal{L}\left\{\frac{\partial^2 u(x,s)}{\partial x^2}\right\} = \frac{d^2\bar{u}(x,t)}{dx^2}$$

substitution $\mathcal{L}\left\{\frac{?u(x,t)}{?x}\right\}$ and $\mathcal{L}\left\{\frac{?^2u(x,s)}{?x^2}\right\}$ into equation (9),

$$s\bar{u}(x,s) - u(x,0) = \frac{d^2\bar{u}(x,s)}{dx^2} \tag{12}$$

Equation (12) is a second-order homogeneous heat equation which is reduced to the ordinary differential equation, so that a general solution is obtained. Then solving equation (12) to get the general equation with the case of a second order homogeneous differential equation with a constant coefficient is as follow :

$$a_0\frac{d^2\bar{u}}{dx^2} + a_1\frac{d\bar{u}}{dx} + a_2\bar{u} = 0 \tag{13}$$

With $a_0, a_1, dan\ a_2$ is assume constant. If $D = \frac{d}{dx}$ get :

$$(a_0D^2 + a_1D + a_2)\bar{u} = 0$$

Then the characteristic roots can be solved with the formula in the quadratic equation, so that the general solution of the differential equation is obtained

$$\bar{u} = e^{-\frac{1}{2a_0}a_1 x}(c_1 \cos \omega x + c_2 \sin \omega x) \quad (14)$$

Equation (14) is a general solution for two complex roots where c_1 and c_2 are the corresponding constants.

By transforming the conditions given and substituted into the general equation obtained, a new general solution $\bar{u}(x,s)$ can be obtained to determine the solution to the equation determining the temperature $u(x,t)$ of a horizontal cylindrical, generated by determining the inverse Laplace transform of $\bar{u}(x,s)$ in obtained by using the inverse Laplace transform table so that it will be obtained $u(x,t)$

CONCLUSION

From the results of the research, it can be concluded that the Laplace transform can reduce the one-dimensional homogeneous heat equation $\frac{?u}{?t} = k\frac{?^2u}{?x^2}$ to be system of differential equations $s\bar{u}(x,s) - u(x,0) = \frac{\partial^2 \bar{u}(x,s)}{\partial x^2}$.

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