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Application Of Transportation Method In Optimizing The Cost Of Equal Distribution Of Zakat Fitrah In Ngaglik District, Sleman, Yogyakarta

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Abstract

Zakat is a form of worship that can bring many benefits to Muslims, especially zakat fitrah. However, several problems remain frequently encountered in the implementation of zakat fitrah, one of which is the distribution of varying amounts of rice in each region. To address this issue, it is necessary to equalize the distribution of zakat fitrah. This can be achieved by distributing zakat fitrah to regions receiving more rice, especially those receiving less. To achieve this, it is necessary to find an effective method to make the distribution process more efficient. One method that can be used is transportation, which will make the allocation of goods more effective in terms of labor, time, and costs. In the case of zakat fitrah, each region acts as both a source and a destination. The source supply is based on the amount of zakat fitrah received, while the destination demand is based on the number of zakat recipients in each region. This study uses data from hamlets in Ngaglik sub-district, Sleman, Yogyakarta. The solution to the equitable distribution of zakat fitrah is solved using a transportation model with the North-West Center method, the Least Cost method, and the Vogel Approach Method (VAM) which is useful for analyzing initial feasibility. The minimum solution of the three methods is obtained by the Least Cost Method; Modified Distribution (MODI) to analyze the optimum solution. From the calculation of the transportation method, the optimum cost is obtained at Rp. 972,350.00; zakat fitrah recipients receive the same amount of 5.6 kg per head of family; and the determination of zakat fitrah delivery from one region to another.

Keywords: Zakat Fitrah, Transportation Methods, Distribution of Zakat Fitrah.

INTRODUCTION

Zakat is an important aspect of Islam, just as the five daily prayers are. Zakat is one of the Five Pillars of Islam and occupies the third position after prayer. When a Muslim fulfills this obligation, their practice of Islam becomes more complete. Zakat is also a form of worship that brings benefits to the Muslim community, including strengthening social bonds among fellow Muslims. Allah commands zakat because this obligation can strengthen the relationship between the wealthy and the poor, thereby fostering unity among them and making them part of one cohesive body (society or nation) (Faizin, 2021). According to Islamic law (*sharia*), zakat refers to a mandatory right that must be paid from one's wealth. Zakat is defined as giving a specific portion of certain wealth that has reached the *nisab* threshold to those who are entitled to receive it. Once ownership has reached the *nisab* and the *haul* (one lunar year of possession) has been completed—except for mined resources and discovered treasures—zakat must be paid immediately (Anis, 2020).

There are two types of zakat: zakat al-fitr and zakat al-mal. Zakat al-fitr refers to zakat associated with staple food, as it is given in the form of food on the day of Eid al-Fitr with the purpose of purifying and cleansing the soul (Varlina, 2021). The payment of zakat al-fitr is obligatory for every Muslim, whether a child or an adult, male, female, or *khuntha* (intersex), free person or slave, and whether mentally competent or not (Khairuddin, 2020).

According to Surah At-Taubah verse 60, there are several categories of people who are entitled to receive zakat: the poor (*fakir*), the needy (*miskin*), zakat administrators (*amil*), new converts to Islam (*muallaf*), those seeking freedom from bondage (*riqab*), debtors (*gharim*), those striving in the cause of Allah (*sabilillah*), and stranded travelers (*ibnu sabil*). A *fakir* is a person who possesses neither wealth nor employment sufficient to meet daily needs. For example, if the minimum basic requirement is ten units, such a person can only fulfill four units or even less (Tuasikal, 2021). In contrast, a *miskin* is someone who owns some wealth, but the amount is insufficient to meet basic needs (Hakim, 2023).

The distribution of zakat al-fitr refers to the allocation or delivery of zakat proceeds to eligible recipients. Zakat distribution has both targets and objectives. The targets are the individuals or groups who are eligible to receive zakat, while the objective is to improve social welfare in the economic sector, thereby reducing the number of underprivileged people and ultimately increasing the number of zakat payers. Zakat distribution is an activity aimed at facilitating and streamlining the transfer of zakat funds from payers (*muzakki*) to recipients (*mustahiq*). The collected funds are distributed through institutions responsible for managing zakat. Through proper distribution, zakat funds can reach the intended recipients accurately and according to their needs. Furthermore, effective distribution ensures that wealth is spread more evenly and does not remain concentrated within certain groups or regions only (Islami, 2021).

The Office of Religious Affairs (*Kantor Urusan Agama* or KUA) is a government office that carries out part of the responsibilities of the Ministry of Religious Affairs, including marriage registration, reconciliation services, mosque management and development, waqf administration, population affairs, the promotion of harmonious families (*keluarga sakinah*), and zakat management, particularly zakat al-fitr. In its daily operations, the KUA manages zakat al-fitr activities, from collection to distribution.

The KUA of Ngaglik, as the implementing agency for the zakat al-fitr distribution program serving recipients in several villages, requires a substantial amount of funding for distribution activities. To minimize distribution costs, careful planning of zakat al-fitr distribution is necessary so that the costs incurred are optimal. One method used to optimize distribution costs is the transportation method.

Table 1. Amount of Rice Collected and Received per Household

Hamlet/Village	Collected Rice (kg)	Zakat Recipients (Households)	Rice Received (per Family Head) (kg)
Banteran	750	171	4
Ngemplak	995	267	4
Jetis Suruh	705	67	11
Penen	792	76	10
Kayunan	535	167	3
Panasan	805	223	4
Donolayan	1,544	201	8
Gondanglutung	1,655	198	8
Maron	277	55	5
Wonolelo	765	150	5
Klidon	2,245	151	15
Sardonoharjo	1,075	216	5
Ploso Kuning IV	1,271	437	3

From the data, it can be seen that the amount of zakat received by beneficiaries varies from one village to another. If all zakat recipients are expected to receive the same amount, it is necessary to implement an equitable distribution of zakat al-fitr. However, achieving such equity requires transportation and distribution costs to transfer zakat al-fitr from one area to another. Therefore, it is necessary to determine the minimum distribution cost.

Optimization is a collection of mathematical formulations and numerical methods used to find and identify the best candidate from a set of alternatives without explicitly calculating and evaluating every possible alternative (Aminullah, 2018). There are various approaches that can be used to solve a problem and obtain the best possible result. These approaches are referred to as optimization systems. Generally, optimization systems refer to mathematical programming techniques that address or are related to operations research programs dealing with the problem at hand (Yunita, 2018). Optimization problems are usually expressed in the form of mathematical functions. Optimization is the process of maximizing or minimizing an objective function while taking existing constraints into consideration (Fachrudin, 2019).

One of the models used in optimization problems is the transportation model. In general, transportation refers to the movement of a product from a collection point (*assembly point*) to a destination with the objective of minimizing the total transportation cost incurred by the transportation provider (Yudhanegara, 2021). A transportation problem involves *m* sources, *n* destinations, and transportation costs associated with moving each unit of product from a source to a destination (L. Winston, 2022). In a transportation problem, the total quantity shipped from all sources is equal to the total quantity received by all destinations.

The transportation model appears to be suitable for addressing the problem of equitable zakat distribution because it aligns with its fundamental definition: moving a product from a collection point to a destination while minimizing transportation costs. In this context, the collection points are areas that receive a surplus of zakat, while the destinations are areas that receive relatively less zakat. The objective is to minimize the costs required to redistribute zakat more evenly among beneficiaries.

Based on the considerations above, the researcher is interested in investigating whether the transportation method can provide cost savings and improve the efficiency of equalizing the distribution of zakat al-fitr.

RESEARCH METHODS

The data used in this research is the number of zakat fitrah institutions established by zakat payers and recipients in several hamlets in Ngaglik, Sleiman. Permit data was obtained from the Office of Religious Affairs (KUA) of Ngaglik Sub-district, located at Meinara Masjid Agung Dr. Wahidin Soeidirohoeisodo, Jl. Parasamya, Beiran, Tridadi, Sleiman, Sleiman Regency, Yogyakarta. Thirteen hamlets/villages in Ngaglik participated in collecting zakat fitrah data at the KUA of Ngaglik Sub-district.

Table 2. Number of Zakat Fitrah Institutions Available in 2023

No.	Zakat Amil (Supply/Demand)	Location	Rice Supply (kg)
1	Al-Bashiroh Mosque	Banteran	750
2	Nurul Iman Mosque	Ngemplak	995
3	Nurul Iman Mosque	Jetis Suruh	705
4	Abdurrahman Al-Eid Mosque	Penen	792
5	Walisongo Mosque	Kayunan	535
6	Panasan Mosque	Panasan	805
7	Al-Ikhlas Islamic Study Assembly (Majelis Taklim)	Donolayan	1,544
8	Baiturrahim Mosque	Gondanglutung	1,655
9	Al-Firdaus Prayer House (Musholla)	Maron	277
10	Wonolelo	Wonolelo, Wonoseto	765
11	Baiturrahman Mosque	Klidon	2,245
12	Sholihiin Mosque	Sardonoharjo	1,075
13	Pathok Negoro Mosque	Plosokuning IV	1,271

Table 3. Shipping Rates from Warehouse to Distribution Point (Rp. 100/Kg)

Dusun	Banteran	Ngemplak	Jetis Suruh	Penen	Kayunan	Panasan	Donolayan	Gondanglutung	Maron	Wonolelo	Klidon	Sardonoharjo	Plosokuning
Banteran	0,0	1,9	3,6	2,0	2,0	0,7	0,8	1,9	3,2	2,7	6,8	6,2	6,5
Ngemplak	1,9	0,0	5,1	0,3	4,7	1,5	2,6	3,7	5,0	5,3	3,7	6,9	5,5
Jetis Suruh	3,6	5,1	0,0	7,2	1,8	3,9	2,7	2,5	0,8	2,3	7,3	6,1	10,2
Penen	2,0	0,3	7,2	0,0	4	1,5	2,8	3,9	5,2	4,7	7,8	7,3	5,2
Kayunan	2,0	4,7	1,8	4	0,0	2,7	1,4	0,7	1,1	0,6	5,6	4,2	7,8
Panasan	0,7	1,5	3,9	1,5	2,7	0,0	1,5	2,6	3,8	3,3	7,1	6,3	6,1
Donolayan	0,8	2,6	2,7	2,8	1,4	1,5	0,0	1,3	2,5	2,0	7,0	5,6	7,1
Gondanglutung	1,9	3,7	2,5	3,9	0,7	2,6	1,3	0,0	1,8	1,3	6,0	4,6	7,5
Maron	3,2	5,0	0,8	5,2	1,1	3,8	2,5	1,8	0,0	1,7	1,8	5,2	9,1
Wonolelo Wonoseto	2,7	5,3	2,3	4,7	0,6	3,3	2,0	1,3	1,7	0,0	5,3	3,9	8,2
Klidon	6,8	3,7	7,3	7,8	5,6	7,1	7,0	6,0	1,8	5,3	0,0	2,0	5,8
Sardonoharjo	6,2	6,9	6,1	7,3	4,2	6,3	5,6	4,6	5,2	3,9	2,0	0,0	5,3
Plosokuning	6,5	5,5	10,2	5,2	7,8	6,1	7,1	7,5	9,1	8,2	5,8	5,3	0

Transportation methods relate to the distribution of a single product from multiple sources with limited supply to multiple destinations with limited demand at minimum distribution cost. Because there is only one type of product, a destination can meet its demand from one or more sources (Lasmana, 2021). To achieve minimum costs, product allocation must be arranged in such a way, because there are differences in allocation costs, both from source to destination and vice versa.

Transportation problems are essentially a class of linear programs that can be solved using the simplex method. However, due to their unique characteristics, transportation problems require more practical and efficient calculation methods (Lasmana, 2021).

Transportation problems have several characteristics, including (Lasmana, 2021):

1. There are a number of sources and a number of destinations.
2. The quantity of goods distributed from each source and demanded by each destination is fixed.
3. The quantity of goods shipped from a source to a destination is equal to the demand or capacity of the source.
4. The transportation cost from a source to a destination is fixed. Mathematically, transportation problems can be modeled as follows:

Objective function:

$$\text{Minimum } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

with obstacles:

$$\sum_{i=1}^{m} X_{ij} = a_i ; i = 1,2, \dots , m$$

$$\sum_{i=1}^{n} X_{ij} = b_j ; j = 1,2, \dots , n$$

Descriptions:

C_{ij} = transportation cost per unit of goods from source i to destination j

X_{ij} = quantity of goods distributed from source i to destination j

a_i = quantity of goods offered or capacity of source i

b_j = quantity of goods requested or ordered by destination j

m = number of sources

n = number of destinations

A transportation problem is said to be balanced (balanced program) if the sum of supply at source *i* equals the sum of demand at destination *j* (Sabrina, 2018).
This can be written as:

$$\sum_{i=1}^{m} a_i = \sum_{i=1}^{n} b_j$$

Transportation problems can be expressed in a special table called a transportation table. Sources are listed in rows and destinations in columns. A transportation table contains m x n boxes. The transportation cost per unit of goods C_{ij} is recorded in the small box at the top right of each box. The demand from each destination is listed in the bottom row, while the supply from each source is recorded in the rightmost column. The bottom left corner box indicates that supply (S) is equal to demand (D). The variable X_{ij} in each box indicates the quantity of goods transported from source *i* to destination *j*. The general form of a transportation table can be seen in Table 2.

Table 4. Transportation Table

Dari \ Ke		Tujuan						Supply
		1	2	...	*j*	...	*n*	
Sumber	1	C_{11} X_{11}	C_{12}		C_{1j}		C_{1n} X_{1n}	S_1
	2	C_{21} X_{21}	C_{22} X_{22}		C_{2j} X_{2j}		C_{2n} X_{2n}	S_2
	⋮	⋮	⋮		⋮		⋮	⋮
	i	C_{i1}	C_{i2}		C_{ij}		C_{in}	S_i
	⋮	⋮	⋮		⋮		⋮	⋮
	m	C_{m1} X_{m1}	C_{m2} X_{m2}		C_{mj} X_{mj}		C_{mn} X_{mn}	S_m
Demand		D_1	D_2		D_j		D_n	$\Sigma S_i = \Sigma D_j$

The first step in solving a transportation problem is to determine an initial feasible solution. There are three methods commonly used to obtain an initial feasible solution:

1. Northwest Corner Method
   The steps of the Northwest Corner Method are as follows:
   a. The initial allocation is assigned to the cell located in the upper-left corner of the transportation table. The value allocated to this cell depends on the supply and demand constraints associated with the cell. Allocate the largest possible value to cell X_{11}while considering the supply and demand constraints. Thus: $X_{11} = \min(S_1, D_1)$
   b. Allocate the largest possible value to the cell adjacent to X_{11}.
   c. Repeat Step 3 until all supply and demand constraints have been satisfied.
2. Least Cost Method
   The steps of the Least Cost Method are as follows:
   a. Select the cell with the lowest transportation cost C_{ij}, then allocate as much supply or demand as possible to that cell. For the smallest C_{ij}, $X_{ij} = \min(S_i, D_j)$. This allocation will exhaust either row $i$or column j. Any row or column whose supply or demand has been completely satisfied is then eliminated from further consideration.

b. From the remaining cells (those not eliminated), select the cell with the next lowest transportation cost C_{ij} and allocate as much as possible to row i or column j.
c. Continue this process until all supply and demand requirements have been fulfilled.

3. Vogel's Approximation Method (VAM)
   The steps of the Vogel's Approximation Method (VAM) are as follows:
   a. Calculate the opportunity cost (penalty) for each row and column. The opportunity cost for a row is obtained by subtracting the smallest transportation cost C_{ij} in that row from the second-smallest cost in the same row. The opportunity cost for a column is calculated similarly. These penalties represent the additional cost incurred by not selecting the minimum-cost cell.
   b. Select the row or column with the largest opportunity cost (if there is a tie, choose arbitrarily). Allocate as many units as possible to the cell with the minimum transportation cost C_{ij} within the selected row or column. Thus, $X_{ij} = \min(S_i, D_j)$. This allocation avoids the largest penalty.
   c. Adjust the supply and demand values to reflect the allocation made. Eliminate any row or column whose supply or demand has been completely satisfied.
   d. If all supply and demand requirements have not yet been met, return to Step 1 and recalculate the opportunity costs. If all supply and demand requirements have been satisfied, an initial feasible solution has been obtained.

After obtaining an initial feasible solution, the next step is to determine the optimal solution. Two methods are commonly used:

1. Stepping Stone Method
2. Modified Distribution Method (MODI)
   The steps of the MODI method are as follows:
   a. Determine the values of U_i for each row and V_j for each column using the relationship: $C_{ij} = U_i + V_j$, for all basic variables. One of the U_i values is arbitrarily set equal to zero.
   b. Calculate the opportunity cost (or reduced cost) for each non-basic variable using: $X_{ij} = C_{ij} - U_i - V_j$.
   c. If there is any negative value of X_{ij}, the current solution is not yet optimal. Select the non-basic variable with the largest negative value as the entering variable.
   d. Allocate units to the entering variable X_{ij} according to the Stepping Stone procedure.
   e. Repeat Steps 1 through 4 until all values of X_{ij} are zero or positive. At that point, the optimal solution has been obtained.

RESULTS AND DISCUSSION

The data obtained are formulated into a mathematical model as follows:

$$\text{Minimum } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Thus,

$$\text{Minimum } Z = \sum_{i=1}^{13} \sum_{j=1}^{13} C_{ij} X_{ij}$$

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where C_{ij} represents the transportation cost from source i to destination j, as presented in Table 4 Subject to the following constraints:

Supply Constraints:

$$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+X_{18}+X_{19}+X_{110}+X_{111}+X_{112}+X_{113}=750$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+X_{28}+X_{29}+X_{210}+X_{211}+X_{212}+X_{213}=995$$

$$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+X_{38}+X_{39}+X_{310}+X_{311}+X_{312}+X_{313}=705$$

$$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+X_{48}+X_{49}+X_{410}+X_{411}+X_{412}+X_{413}=792$$

$$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+X_{58}+X_{59}+X_{510}+X_{511}+X_{512}+X_{513}=535$$

$$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+X_{68}+X_{69}+X_{610}+X_{611}+X_{612}+X_{613}=805$$

$$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+X_{78}+X_{79}+X_{710}+X_{711}+X_{712}+X_{713}=1544$$

$$X_{81}+X_{82}+X_{83}+X_{84}+X_{85}+X_{86}+X_{87}+X_{88}+X_{89}+X_{810}+X_{811}+X_{812}+X_{813}=1655$$

$$X_{91}+X_{92}+X_{93}+X_{94}+X_{95}+X_{96}+X_{97}+X_{98}+X_{99}+X_{910}+X_{911}+X_{912}+X_{913}=277$$

$$X_{101}+X_{102}+X_{103}+X_{104}+X_{105}+X_{106}+X_{107}+X_{108}+X_{109}+X_{1010}+X_{1011}+X_{1012}+X_{1013}=765$$

$$X_{111}+X_{112}+X_{113}+X_{114}+X_{115}+X_{116}+X_{117}+X_{118}+X_{119}+X_{1110}+X_{1111}+X_{1112}+X_{1113}=2245$$

$$X_{121}+X_{122}+X_{123}+X_{124}+X_{125}+X_{126}+X_{127}+X_{128}+X_{129}+X_{1210}+X_{1211}+X_{1212}+X_{1213}=1075$$

$$X_{131}+X_{132}+X_{133}+X_{134}+X_{135}+X_{136}+X_{137}+X_{138}+X_{139}+X_{1310}+X_{1311}+X_{1312}+X_{1313}=1271$$

Request:

$$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+X_{61}+X_{71}+X_{81}+X_{91}+X_{101}+X_{111}+X_{121}+X_{131}=964$$

$$X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+X_{62}+X_{72}+X_{82}+X_{92}+X_{102}+X_{112}+X_{122}+X_{132}=1506$$

$$X_{13}+X_{23}+X_{33}+X_{43}+X_{53}+X_{63}+X_{73}+X_{83}+X_{93}+X_{103}+X_{113}+X_{123}+X_{133}=378$$

$$X_{14}+X_{24}+X_{34}+X_{44}+X_{54}+X_{64}+X_{74}+X_{84}+X_{94}+X_{104}+X_{114}+X_{124}+X_{134}=429$$

$$X_{15}+X_{25}+X_{35}+X_{45}+X_{55}+X_{65}+X_{75}+X_{85}+X_{95}+X_{105}+X_{115}+X_{125}+X_{135}=942$$

$$X_{16}+X_{26}+X_{36}+X_{46}+X_{56}+X_{66}+X_{76}+X_{86}+X_{96}+X_{106}+X_{116}+X_{126}+X_{136}=1257$$

$$X_{17}+X_{27}+X_{37}+X_{47}+X_{57}+X_{67}+X_{77}+X_{87}+X_{97}+X_{107}+X_{117}+X_{127}+X_{137}=1133$$

$$X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78} + X_{88} + X_{98} + X_{108} + X_{118} + X_{128} + X_{138} = 1116$$

$$X_{19} + X_{29} + X_{39} + X_{49} + X_{59} + X_{69} + X_{79} + X_{89} + X_{99} + X_{109} + X_{119} + X_{129} + X_{139} = 310$$

$$X_{110} + X_{210} + X_{310} + X_{410} + X_{510} + X_{610} + X_{710} + X_{810} + X_{910} + X_{1010} + X_{1110} + X_{1210} + X_{1310} = 846$$

$$X_{111} + X_{211} + X_{311} + X_{411} + X_{511} + X_{611} + X_{711} + X_{811} + X_{911} + X_{1011} + X_{1111} + X_{1211} + X_{1311} = 851$$

$$X_{121} + X_{212} + X_{312} + X_{412} + X_{512} + X_{612} + X_{712} + X_{812} + X_{912} + X_{1012} + X_{1112} + X_{1212} + X_{1312} = 1218$$

$$X_{131} + X_{213} + X_{313} + X_{413} + X_{513} + X_{613} + X_{713} + X_{813} + X_{913} + X_{1013} + X_{1113} + X_{1213} + X_{1313} = 2464$$

Northwest Corner Method (NWC)

Compare the values of supply and demand.

The smaller of the two values is allocated to cell X_{11}. This allocation exhausts the available capacity of the corresponding supply (or demand) and leaves the remaining balance in column D_1(or the respective row/column). The first stage of the allocation process is presented in Table 5.

Table 5. First Iteration of the Northwest Corner Method (NWC)

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 **750**	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	0
S_2	1.9	0	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5	995
S_3	3.6	5.1	0	7.2	1.8	3.9	2.7	2.5	0.8	2.3	7.3	6.1	10.2	705
S_4	2	0.3	7.2	0	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2	792
S_5	2	4.7	1.8	4	0	2.7	1.4	0.7	1.1	0.6	5.6	4.2	7.8	535
S_6	0.7	1.5	3.9	1.5	2.7	0	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805
S_7	0.8	2.6	2.7	2.8	1.4	1.5	0	1.3	2.5	2	7	5.6	7.1	1544
S_8	1.9	3.7	2.5	3.9	0.7	2.6	1.3	0	1.8	1.3	6	4.6	7.5	1655
S_9	3.2	5	0.8	5.2	1.1	3.8	2.5	1.8	0	1.7	1.8	5.2	9.1	277
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	0	5.3	3.9	8.2	765
S_{11}	6.8	3.7	7.3	7.8	5.6	7.1	7	6	1.8	5.3	0	2	5.8	2245
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0	5.3	1075
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	0	1271
Demand	214	1506	378	429	942	1257	1133	1116	310	846	851	218	2464	

The iteration phase continues by repeating the steps of the NWC method. The initial solution is obtained by conducting twenty-five iterations. The initial solution for allocating the NWC can be seen in Table 6.

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Table 6. Initial Solution for Allocating the NWC

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	**750** 0	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	750
S_2	**214** 1.9	**781** 0	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5	995
S_3	3.6	**705** 5.1	0	7.2	1.8	3.9	2.7	2.5	0.8	2.3	7.3	6.1	10.2	705
S_4	2	**20** 0.3	**378** 7.2	**394** 0	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2	792
S_5	2	4.7	1.8	**35** 4	**500** 0	2.7	1.4	0.7	1.1	0.6	5.6	4.2	7.8	535
S_6	0.7	1.5	3.9	1.5	**442** 2.7	**363** 0	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805
S_7	0.8	2.6	2.7	2.8	1.4	**894** 1.5	**650** 0	1.3	2.5	2	7	5.6	7.1	1544
S_8	1.9	3.7	2.5	3.9	0.7	2.6	**483** 1.3	**1116** 0	**56** 1.8	1.3	6	4.6	7.5	1655
S_9	3.2	5	0.8	5.2	1.1	3.8	2.5	1.8	**254** 0	23 1.7	1.8	5.2	9.1	277
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	**765** 0	5.3	3.9	8.2	765
S_{11}	6.8	3.7	7.3	7.8	5.6	7.1	7	6	1.8	**58** 5.3	**851** 0	**1218** 2	**118** 5.8	2245
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0	**1075** 5.3	1075
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	**1271** 0	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	218	2464	

The minimum transportation cost using the NWC method is:

$$\begin{aligned} Min.Z = {} & 0 \times 750 + 1.9 \times 214 + 0 \times 781 + 5.1 \times 705 + 0.3 \times 20 + 7.2 \times 378 + 0 \times 394 \\ & + 4 \times 35 + 0 \times 500 + 2.7 \times 442 + 0 \times 363 + 1.5 \times 894 + 0 \times 650 + 1.3 \times 483 \\ & + 0 \times 1116 + 1.8 \times 56 + 0 \times 254 + 1.7 \times 23 + 0 \times 765 + 5.3 \times 58 + 0 \times 851 \\ & + 2 \times 1218 + 5.8 \times 118 + 5.3 \times 1075 + 0 \times 1271 = 19297.2 \end{aligned}$$

Least Cost Method

The lowest transportation cost is 0, which is located in cell X_{ij}. The first stage of the allocation process is presented in Table 7.

Table 7. First Iteration of the Least Cost Method

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 750	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	750
S_2	1.9	0	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5	995
S_3	3.6	5.1	0	7.2	1.8	3.9	2.7	2.5	0.8	2.3	7.3	6.1	10.2	705
S_4	2	0.3	7.2	0	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2	792
S_5	2	4.7	1.8	4	0	2.7	1.4	0.7	1.1	0.6	5.6	4.2	7.8	535
S_6	0.7	1.5	3.9	1.5	2.7	0	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805
S_7	0.8	2.6	2.7	2.8	1.4	1.5	0	1.3	2.5	2	7	5.6	7.1	1544
S_8	1.9	3.7	2.5	3.9	0.7	2.6	1.3	0	1.8	1.3	6	4.6	7.5	1655
S_9	3.2	5	0.8	5.2	1.1	3.8	2.5	1.8	0	1.7	1.8	5.2	9.1	277
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	0	5.3	3.9	8.2	765
S_{11}	6.8	3.7	7.3	7.8	5.6	7.1	7	6	1.8	5.3	0	2	5.8	2245
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0	5.3	1075
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	0	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	218	2464	

The iteration phase continues by repeating the steps of the Least Cost method. The initial solution is obtained by conducting twenty-five iterations. The initial solution for allocating Least Cost can be seen in Table 8.

Table 8. Initial Solution for Allocating Least Cost

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 **750**	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	750
S_2	1.9	0 **995**	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5	995
S_3	3.6	5.1	0 **378**	7.2	1.8	3.9 **204**	2.7	2.5	0.8 33	2.3	7.3	6.1	10.2 **90**	705
S_4	2	0.3 **363**	7.2	0 **429**	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2	792
S_5	2	4.7	1.8	4	0 **535**	2.7	1.4	0.7	1.1	0.6	5.6	4.2	7.8	535
S_6	0.7	1.5	3.9	1.5	2.7	0 **805**	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805
S_7	0.8 214	2.6	2.7	2.8	1.4	1.5 **197**	0 1133	1.3	2.5	2	7	5.6	7.1	1544
S_8	1.9	3.7	2.5	3.9	0.7 **407**	2.6 **51**	1.3	0 **1116**	1.8	1.3 **81**	6	4.6	7.5	1655
S_9	3.2	5	0.8	5.2	1.1	3.8	2.5	1.8	0 **277**	1.7	1.8	5.2	9.1	277
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	0 **765**	5.3	3.9	8.2	765
S_{11}	6.8	3.7 **148**	7.3	7.8	5.6	7.1	7	6	1.8	5.3	0 **851**	2 **143**	5.8 **1103**	2245
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0 **1075**	5.3	1075
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	0 **1271**	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	218	2464	

The minimum transportation cost using the Least Cost method is:

$$\begin{aligned} Min. Z = \ & 0 \times 750 + 0 \times 995 + 0 \times 378 + 3.9 \times 204 + 0.8 \times 33 + 10.2 \times 90 + 0.3 \times 363 \\ & + 0 \times 429 + 0 \times 535 + 0 \times 805 + 0.8 \times 214 + 1.5 \times 197 + 0 \times 1133 \\ & + 0.7 \times 407 + 2.6 \times 51 + 0 \times 1116 + 1.3 \times 81 + 0 \times 277 + 0 \times 765 + 3.7 \times 148 \\ & + 0 \times 851 + 2 \times 143 + 5.8 \times 1103 + 0 \times 1075 + 0 \times 1271 = 10069.4 \end{aligned}$$

VAM Method

The initial feasible solution using the VAM method can be seen in Table 9.

Table 9. First Stage of VAM Iteration

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply	Penalti Baris
S_1	0	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	750	0.7
S_2	1.9	0	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5	995	0.3
S_3	3.6	5.1	0	7.2	1.8	3.9	2.7	2.5	0.8	2.3	7.3	6.1	10.2	705	0.8
S_4	2	0.3	7.2	0	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2	792	0.3
S_5	2	4.7	1.8	4	0	2.7	1.4	0.7	1.1	0.6	5.6	4.2	7.8	535	0.6
S_6	0.7	1.5	3.9	1.5	2.7	0	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805	0.7
S_7	0.8	2.6	2.7	2.8	1.4	1.5	0	1.3	2.5	2	7	5.6	7.1	1544	0.8
S_8	1.9	3.7	2.5	3.9	0.7	2.6	1.3	0	1.8	1.3	6	4.6	7.5	1655	0.7
S_9	3.2	5	0.8	5.2	1.1	3.8	2.5	1.8	0	1.7	1.8	5.2	9.1	277	0.8
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	0	5.3	3.9	8.2	765	0.6
S_{11}	6.8	3.7	7.3	7.8	5.6	7.1	7	6	1.8	5.3	0	2	5.8	2245	1.8
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0	5.3	1075	2
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	0 1271	1271	5.2
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	218	2464		
Penalti Kolom	0.7	0.3	0.8	0.3	0.6	0.7	0.8	0.7	0.8	0.6	1.8	2	5.2		

The iteration phase continues by repeating the steps of the VAM method. The initial solution is obtained by conducting twenty-five iterations. The initial solution for the VAM allocation can be seen in Table 10.

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Table 10. Initial Solution for the VAM Allocation

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 **750**	1.9	3.6	2	2	0.7	0.8	1.9	3.2	2.7	6.8	6.2	6.5	750
S_2	1.9	0 **362**	5.1	0.3	4.7	1.5	2.6	3.7	5	5.3	3.7	6.9	5.5 **633**	995
S_3	3.6	5.1 **203**	0 **378**	7.2	1.8	3.9 **124**	2.7	2.5	0.8	2.3	7.3	6.1	10.2	705
S_4	2	0.3	7.2	0 **429**	4	1.5	2.8	3.9	5.2	4.7	7.8	7.3	5.2 **363**	792
S_5	2	4.7	1.8	4	0 **403**	2.7 **51**	1.4	0.7	1.1	0.6 **81**	5.6	4.2	7.8	535
S_6	0.7	1.5	3.9	1.5	2.7	0 **805**	1.5	2.6	3.8	3.3	7.1	6.3	6.1	805
S_7	0.8 214	2.6	2.7	2.8	1.4	1.5	0 **1133**	1.3	2.5	2	7	5.6	7.1 **197**	1544
S_8	1.9	3.7	2.5	3.9	0.7 **539**	2.6	1.3	0 **1116**	1.8	1.3	6	4.6	7.5	1655
S_9	3.2	5	0.8	5.2	1.1	3.8 **277**	2.5	1.8	0	1.7	1.8	5.2	9.1	277
S_{10}	2.7	5.3	2.3	4.7	0.6	3.3	2	1.3	1.7	0 **765**	5.3	3.9	8.2	765
S_{11}	6.8	3.7 **941**	7.3	7.8	5.6	7.1	7	6	1.8 **310**	5.3	0 **851**	2 **143**	5.8	2245
S_{12}	6.2	6.9	6.1	7.3	4.2	6.3	5.6	4.6	5.2	3.9	2	0 **1075**	5.3	1075
S_{13}	6.5	5.5	10.2	5.2	7.8	6.1	7.1	7.5	9.1	8.2	5.8	5.3	0 **1271**	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	218	2464	

The minimum transportation cost using the VAM method is:

$$\begin{aligned} Min. Z = {} & 0 \times 750 + 0 \times 362 + 5.5 \times 633 + 5.1 \times 203 + 0 \times 378 + 3.9 \times 124 + 0 \times 429 \\ & + 5.2 \times 363 + 0 \times 403 + 2.7 \times 51 + 0.6 \times 81 + 0 \times 805 + 0.8 \times 214 + 0 \times 1133 \\ & + 7.1 \times 197 + 0.7 \times 539 + 0 \times 1116 + 3.8 \times 277 + 0 \times 765 + 3.7 \times 941 \\ & + 1.8 \times 310 + 0 \times 851 + 2 \times 143 + 0 \times 1075 + 0 \times 1271 = 14399.8 \end{aligned}$$

The minimum transportation cost obtained from the three methods above was achieved using the Least Cost Method. Therefore, the optimal solution will be determined using the initial allocation obtained from the Least Cost Method.

The initial solution generated by the Least Cost Method is then re-evaluated using the Modified Distribution Method (MODI) to obtain the optimal solution. The first step is to determine the row values (U_i) and column values $\left(V_j\right)$ for each basic variable using the relationship: $C_{ij} = U_i + V_j$, where C_{ij} represents the transportation cost, and one of the U_i values is set equal to zero. This relationship is then used to calculate the remaining U_i and V_j values for all basic variables in the transportation tableau.

$$X_{11} = U_1 + V_1 = 0, if\ U_1 = 0\ obtained\ V_1 = 0.$$

$$X_{71} = U_7 + V_1 = 0{,}8, if\ V_1 = 0\ obtained\ U_7 = 0{,}8.$$

$$X_{77} = U_7 + V_7 = 0, if\ U_7 = 0{,}8\ obtained\ V_7 = -0{,}8.$$

$$X_{76} = U_7 + V_6 = 1{,}5, if\ U_7 = 0{,}8\ obtained\ V_6 = 0{,}7.$$

$$X_{66} = U_6 + V_6 = 0, if\ V_6 = 0{,}7\ obtained\ U_6 = -0{,}7.$$

$$X_{36} = U_3 + V_6 = 3{,}9, if\ V_6 = 0{,}7\ obtained\ U_3 = 3{,}2.$$

$$X_{33} = U_3 + V_3 = 0, if\ U_3 = 3{,}2\ obtained\ V_3 = -3{,}2.$$

$$X_{313} = U_3 + V_{13} = 10{,}2, if\ U_3 = 3{,}2\ obtained\ V_{13} = 7.$$

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$$X_{1113} = U_{11} + V_{13} = 5{,}8, if\ V_{13} = 7\ obtained\ U_{11} = -1.2.$$

$$X_{1112} = U_{11} + V_{12} = 2, if\ U_{11} = -1.2\ obtained\ V_{12} = 3{,}2.$$

$$X_{1111} = U_{11} + V_{11} = 0, if\ U_{11} = -1.2\ obtained\ V_{11} = 1{,}2.$$

$$X_{112} = U_{11} + V_{2} = 3{,}7, if\ U_{11} = -1.2\ obtained\ V_{2} = 4{,}9.$$

$$X_{22} = U_{2} + V_{2} = 0, if\ V_{2} = 4{,}9\ obtained\ U_{2} = -4{,}9.$$

$$X_{42} = U_{4} + V_{2} = 0{,}3, if\ V_{2} = 4{,}9\ obtained\ U_{4} = -4{,}6.$$

$$X_{44} = U_{4} + V_{4} = 0, if\ U_{4} = -4{,}6\ obtained\ V_{4} = 4{,}6.$$

$$X_{1212} = U_{12} + V_{11} = 0, if\ V_{11} = 1{,}2\ obtained\ U_{12} = -1{,}2.$$

$$X_{1313} = U_{13} + V_{13} = 0, if\ V_{13}\ obtained\ U_{13} = -7.$$

$$X_{86} = U_{8} + V_{6} = 2{,}6, if\ V_{6} = 0{,}7\ obtained\ U_{8} = 1{,}9.$$

$$X_{88} = U_{8} + V_{8} = 0, if\ U_{8} = 1{,}9\ obtained\ V_{8} = -1{,}9.$$

$$X_{85} = U_{8} + V_{5} = 0{,}7, if\ U_{8} = 1{,}9\ obtained\ V_{5} = -1{,}2.$$

$$X_{55} = U_{5} + V_{5} = 0, if\ V_{5} = -1{,}2\ obtained\ U_{5} = 1{,}2.$$

$$X_{810} = U_{8} + V_{10} = 1{,}3, if\ U_{8} = 1{,}9\ obtained\ V_{10} = -0{,}6.$$

$$X_{1010} = U_{10} + V_{10} = 0, if\ V_{10} = -0{,}6\ obtained\ U_{10} = 0{,}6.$$

$$X_{39} = U_{3} + V_{9} = 0{,}8, if\ U_{3} = 3{,}2\ obtained\ V_{9} = -2{,}4.$$

$$X_{99} = U_{9} + V_{9} = 0, if\ V_{9} = -2{,}4\ obtained\ U_{9} = 2{,}4.$$

After determining the row values and column values, the next step is to calculate the cost change (reduced cost) for each non-basic variable using the reduced cost formula: $X_{ij} = C_{ij} - U_i - V_j$, where $X_{ij}$is a non-basic variable.

The reduced cost value $\Delta_{ij}$for each non-basic variable is presented in the following table:

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Table 11. Reduced Costs X_{ij}

X		j												
		1	2	3	4	5	6	7	8	9	10	11	12	13
i	1	-	-3	3,6	2	3,2	0	1,6	3,8	5,6	3,3	5,6	3	-0,5
	2	6,8	-	5,1	0,3	10,8	5,7	8,3	10,5	12,3	10,8	7,4	8,6	3,4
	3	0,4	-3	-	7,2	-0,2	-	0,3	1,2	-	-0,3	2,9	-0,3	-
	4	6,6	-	7,2	-	9,8	5,4	8,2	10,4	12,2	9,9	11,2	8,7	2,8
	5	0,8	-1,4	1,8	4	-	0,8	1	1,4	2,3	0	3,2	-0,2	-0,4
	6	1,4	-2,7	3,9	1,5	4,6	-	3	5,2	6,9	4,6	6,6	3,8	-0,2
	7	-	-3,1	2,7	2,8	1,8	-	-	2,4	4,1	1,8	5	1,6	-0,7
	8	0	-3,1	2,5	3,9	-	-	0,2	-	2,3	-	2,9	-0,5	-1,4
	9	0,8	-2,3	0,8	5,2	-0,1	0,7	0,9	1,3	-	-0,1	-1,8	-0,4	-0,3
	10	2,1	-0,2	2,3	4,7	1,2	2	2,2	2,6	3,5	-	3,5	0,1	0,6
	11	8	-	7,3	7,8	8	7,6	9	9,1	5,4	7,1	-	-	-
	12	9,4	5,2	6,1	7,3	8,6	8,8	9,6	9,7	10,8	7,7	4	-	1,5
	13	13,5	7,6	10,2	5,2	16	12,4	14,9	16,4	18,5	15,8	11,6	9,1	-

From Table 11, it can be seen that the reduction cost with the negative value is $X_{72} = -3{,}1$. Therefore, X_{72} is included as a basic variable. Then the form of the path is as follows.

Table 12. Creating a Closed Path

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	750 [0]	[1,9]	[3,6]	[2]	[2]	[0,7]	[0,8]	[1,9]	[3,2]	[2,7]	[6,8]	[6,2]	[6,5]	750
S_2	[1,9]	995 [0]	[5,1]	[0,3]	[4,7]	[1,5]	[2,6]	[3,7]	[5]	[5,3]	[3,7]	[6,9]	[5,5]	995
S_3	[3,6]	[5,1]	378 [0]	[7,2]	[1,8]	204 + [3,9]	[2,7]	[2,5]	33 [0,8]	[2,3]	[7,3]	[6,1]	90 − [10,2]	705
S_4	[2]	363 [0,3]	[7,2]	429 [0]	[4]	[1,5]	[2,8]	[3,9]	[5,2]	[4,7]	[7,8]	[7,3]	[5,2]	792
S_5	[2]	[4,7]	[1,8]	[4]	535 [0]	[2,7]	[1,4]	[0,7]	[1,1]	[0,6]	[5,6]	[4,2]	[7,8]	535
S_6	[0,7]	[1,5]	[3,9]	[1,5]	[2,7]	805 [0]	[1,5]	[2,6]	[3,8]	[3,3]	[7,1]	[6,3]	[6,1]	805
S_7	214 [0,8]	+ [2,6]	[2,7]	[2,8]	[1,4]	− 197 [1,5]	1133 [0]	[1,3]	[2,5]	[2]	[7]	[5,6]	[7,1]	1544
S_8	[1,9]	[3,7]	[2,5]	[3,9]	407 [0,7]	51 [2,6]	[1,3]	1116 [0]	[1,8]	81 [1,3]	[6]	[4,6]	[7,5]	1655
S_9	[3,2]	[5]	[0,8]	[5,2]	[1,1]	[3,8]	[2,5]	[1,8]	277 [0]	[1,7]	[1,8]	[5,2]	[9,1]	277
S_{10}	[2,7]	[5,3]	[2,3]	[4,7]	[0,6]	[3,3]	[2]	[1,3]	[1,7]	765 [0]	[5,3]	[3,9]	[8,2]	765
S_{11}	[6,8]	148 − [3,7]	[7,3]	[7,8]	[5,6]	[7,1]	[7]	[6]	[1,8]	[5,3]	851 [0]	143 [2]	1103 + [5,8]	2245
S_{12}	[6,2]	[6,9]	[6,1]	[7,3]	[4,2]	[6,3]	[5,6]	[4,6]	[5,2]	[3,9]	[2]	1075 [0]	[5,3]	1075
S_{13}	[6,5]	[5,5]	[10,2]	[5,2]	[7,8]	[6,1]	[7,1]	[7,5]	[9,1]	[8,2]	[5,8]	[5,3]	1271 [0]	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	1218	2464	

From Table 12, it can be seen that the symbol "-" with the smallest value, namely the number S_3D_{13}, will be the variable that comes out and becomes a non-basic variable. The results of the first improvement using the MODI method can be seen in Table 13.

Table 13. First Iteration of Improved Analysis

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 **750**	1,9	3,6	2	2	0,7	0,8	1,9	3,2	2,7	6,8	6,2	6,5	750
S_2	1,9	0 **995**	5,1	0,3	4,7	1,5	2,6	3,7	5	5,3	3,7	6,9	5,5	995
S_3	3,6	5,1	0 **378**	7,2	1,8	3,9 **294**	2,7	2,5	0,8 **33**	2,3	7,3	6,1	10,2	705
S_4	2	0,3 **363**	7,2	0 **429**	4	1,5	2,8	3,9	5,2	4,7	7,8	7,3	5,2	792
S_5	2	4,7	1,8	4	0 **535**	2,7	1,4	0,7	1,1	0,6	5,6	4,2	7,8	535
S_6	0,7	1,5	3,9	1,5	2,7	0 **805**	1,5	2,6	3,8	3,3	7,1	6,3	6,1	805
S_7	0,8 **214**	2,6 **90**	2,7	2,8	1,4	1,5 **107**	0 1133	1,3	2,5	2	7	5,6	7,1	1544
S_8	1,9	3,7	2,5	3,9	0,7 **407**	2,6 **51**	1,3	0 **1116**	1,8	1,3 **81**	6	4,6	7,5	1655
S_9	3,2	5	0,8	5,2	1,1	3,8	2,5	1,8	0 **277**	1,7	1,8	5,2	9,1	277
S_{10}	2,7	5,3	2,3	4,7	0,6	3,3	2	1,3	1,7	0 **765**	5,3	3,9	8,2	765
S_{11}	6,8	3,7 **58**	7,3	7,8	5,6	7,1	7	6	1,8	5,3	0 **851**	2 **143**	5,8 **1193**	2245
S_{12}	6,2	6,9	6,1	7,3	4,2	6,3	5,6	4,6	5,2	3,9	2	0 **1075**	5,3	1075
S_{13}	6,5	5,5	10,2	5,2	7,8	6,1	7,1	7,5	9,1	8,2	5,8	5,3	0 **1271**	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	1218	2464	

The iteration stage continues by repeating the steps of the MODI method until all non-basic X_{ij} values are positive. The optimized solution is obtained by performing three iterations. The optimized solution using MODI is shown in Table 14, and the non-basic X_{ij} values after the third iteration are shown in Table 15.

Table 14. Optimized Solution Using MODI

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	Supply
S_1	0 **750**	1,9	3,6	2	2	0,7	0,8	1,9	3,2	2,7	6,8	6,2	6,5	750
S_2	1,9	0 **995**	5,1	0,3	4,7	1,5	2,6	3,7	5	5,3	3,7	6,9	5,5	995
S_3	3,6	5,1	0 **378**	7,2	1,8 **213**	3,9	2,7	2,5	0,8 **33**	2,3 **81**	7,3	6,1	10,2	705
S_4	2	0,3 **363**	7,2	0 **429**	4	1,5	2,8	3,9	5,2	4,7	7,8	7,3	5,2	792
S_5	2	4,7	1,8	4	0 **535**	2,7	1,4	0,7	1,1	0,6	5,6	4,2	7,8	535
S_6	0,7	1,5	3,9	1,5	2,7	0 **805**	1,5	2,6	3,8	3,3	7,1	6,3	6,1	805
S_7	0,8 **214**	2,6 **90**	2,7	2,8	1,4	1,5 **107**	0 **1133**	1,3	2,5	2	7	5,6	7,1	1544
S_8	1,9	3,7	2,5	3,9	0,7 **194**	2,6 **345**	1,3	0 **1116**	1,8	1,3	6	4,6	7,5	1655
S_9	3,2	5	0,8	5,2	1,1	3,8	2,5	1,8	0 **277**	1,7	1,8	5,2	9,1	277
S_{10}	2,7	5,3	2,3	4,7	0,6	3,3	2	1,3	1,7	0 **765**	5,3	3,9	8,2	765
S_{11}	6,8	3,7 **58**	7,3	7,8	5,6	7,1	7	6	1,8	5,3	0 **851**	2 **143**	5,8 **1193**	2245
S_{12}	6,2	6,9	6,1	7,3	4,2	6,3	5,6	4,6	5,2	3,9	2	0 **1075**	5,3	1075
S_{13}	6,5	5,5	10,2	5,2	7,8	6,1	7,1	7,5	9,1	8,2	5,8	5,3	0 **1271**	1271
Demand	964	1506	378	429	942	1257	1133	1116	310	846	851	1218	2464	

Table 15. X_{ij} **Value Seiteilah Third Iteration**

X		j												
		1	2	3	4	5	6	7	8	9	10	11	12	13
i	1	-	0,1	3,6	2	3,2	0	1,6	3,8	5,4	3,4	8,7	6,1	2,6
	2	3,7	-	5,1	0,3	7,7	2,6	5,2	7,4	9	7,8	7,4	8,6	3,4
	3	0,6	0,3	-	7,2	-	0,2	0,5	1,4	-	-	6,2	3	3,3
	4	3,5	-	7,2	-	6,7	2,3	5,1	7,3	8,9	6,9	11,2	8,7	2,8
	5	0,8	1,7	1,8	4	-	0,8	1	1,4	2,1	0,1	6,3	2,9	2,7
	6	1,4	0,4	3,9	1,5	4,6	-	3	5,2	6,7	4,7	9,7	6,9	2,9
	7	-	-	2,7	2,8	1,8	-	-	2,4	3,9	1,9	8,1	4,7	2,4
	8	0	0	2,5	3,9	-	-	0,2	-	2,1	0,1	6	2,6	1,7
	9	1	1	0,8	5,2	0,1	0,9	1,1	1,5	-	0,2	1,5	2,9	3
	10	2	2,8	2,3	4,7	1,1	1,9	2,1	2,5	3,2	-	6,5	3,1	3,6
	11	4,9	-	7,3	7,8	4,9	4,5	5,9	6	2,1	4,1	-	-	-
	12	6,3	5,2	6,1	7,3	5,5	5,7	6,5	6,6	7,5	4,7	4	-	1,5
	13	10,4	7,6	10,2	5,2	12,9	9,3	11,8	13,3	15,2	12,8	11,6	9,1	-

It can be seen from table 15, the calculation using the MODI method shows that all non-basic variable values are positive, so it can be said that the initial feasible solution obtained using Least Cost is optimal. Then the total cost is calculated using the formula:

$$\text{Minimum } Z = \sum_{i=1}^{13} \sum_{j=1}^{13} C_{ij} X_{ij}$$

therefore:

$$\begin{aligned} Z_{min} = {} & \{(750)(0)\} + \{(995)(0)\} + \{(378)(0)\} + \{(213)(1{,}8)\} + \{(33)(0{,}8)\} \\ & + \{(81)(2{,}3)\} + \{(363)(0{,}3)\} + \{(429)(0)\} + \{(535)(0)\} + \{(805)(0)\} \\ & + \{(214)(0{,}8)\} + \{(90)(2{,}6)\} + \{(107)(1{,}5)\} + \{(1133)(0)\} \\ & + \{(194)(0{,}7)\} + \{(345)(2{,}6)\} + \{(1116)(0)\} + \{(277)(0)\} + \{(765)(0)\} \\ & + \{(58)(3{,}7)\} + \{(851)(0)\} + \{(143)(2)\} + \{(1193)(5{,}8)\} + \{(1075)(0)\} \\ & + \{(1271)(0)\} \\ = {} & 383{,}4 + 26{,}4 + 186{,}3 + 108{,}9 + 171{,}2 + 234 + 160{,}5 + 135{,}8 + 897 + 214{,}6 + 286 \\ & + 6919{,}4 \\ = {} & 9723{,}5 \end{aligned}$$

Therefore, the transportation cost for the average distribution of zakat to 13 hamlets in Ngaglik, Sleiman, is a minimum of Rp 972,350.00, with distribution details as shown in Table 16 below.

Table 16. Details of Zakat Fitrah Distribution to Minimize Transportation Costs

No.	Zakat Amil	Distribution Point	Amount of Rice (kg)
1	Penen	Ngemplak	363
2	Donolayan	Banteran	214
3	Donolayan	Ngemplak	90
4	Klidon	Ngemplak	58
5	Jetis Suruh	Kayunan	213
6	Gondanglutung	Kayunan	194
7	Gondanglutung	Panasan	345
8	Donolayan	Panasan	107
9	Jetis Suruh	Maron	33
10	Jetis Suruh	Wonolelo	81
11	Klidon	Sardonoharjo	143
12	Klidon	Plosokuning IV	1,193

CONCLUSIONS

The following conclusions can be drawn from the discussion:

1. Using the Least Cost and MODI methods, the optimum cost for equitable distribution of zakat fitrah was Rp 972,350.00. Therefore, transportation is a suitable tool for equitable distribution of zakat.
2. The optimum cost obtained from zakat distribution is shown in Table 12. This indicates that transportation also contributes to the allocation of zakat fitrah rice distribution in achieving equitable distribution of zakat.

In practice, the required data is very difficult and time-consuming. Therefore, it is recommended to write a paper explaining how to collect the data to minimize the time required, allowing for more timely equitable distribution and allocation of zakat fitrah.

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